# Vector Properties of Radially Self-Accelerating Beams

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**Abstract:** We present a complete and consistent theory of scalar and vector radially self-accelerating beams. We show, that only circularly polarized RSABs maintain their self-accelerating character upon focusing, and we discuss their global and local linear and angular momentum content.

Accelerating beams, i.e., electromagnetic fields, that propagate along curved trajectories in free space, without the aid of an external force, have been the subject of an extensive research in the last years. Among the different classes of accelerating beams [1-3], radially self-accelerating beams (RSABs) propagate along spiraling trajectories around their propagation direction [4]. Because of this special property, RSABs have potential impactful applications in different areas of physics, such as sensing [5], material processing [6], and particle manipulation [7]. Despite their high potential, RSABs have only been studied within the scalar electromagnetic theory, and a consistent theory for vector RSABs has not been presented yet. Here, we present a complete theory of vector RSABs, in terms of their linear and angular momentum content. Moreover, we also study under which conditions, the self-accelerating character of RSABs is preserved under focussing.

Scalar RSABs are defined in terms of superpositions of Bessel beams, where each single component is characterized by an angular velocity, proportional to its orbital angular momentum [4]. To generalize their definition to vector beams, we use the Hertz method of potentials to construct electric and magnetic fields, i.e., $\boldsymbol{E}(r,t) = -\partial_t[\nabla\times(\psi_{RSAB}\boldsymbol{e}^{-i\omega t}\boldsymbol{u})]$, and $\boldsymbol{B}(r,t) = \nabla\times[\nabla\times(\psi_{RSAB}\boldsymbol{e}^{-i\omega t}\boldsymbol{u})]$, where $\psi_{\text{RSAB}}$ is the scalar RSAB, given explicitly in Ref. [4], and $\mathbf{u}$ accounts for the initial polarization of the vector beam. To check, whether or not these vector fields are self-accelerating, we apply the criteria developed in Ref. [4], and in particular, we check, whether or not there exists a suitable co-moving reference frame, in which the electric and magnetic fields defined above appear propagation invariant, i.e., $\partial_z\boldsymbol{E}(r,t) = 0 = \partial_z\boldsymbol{B}(r,t)$ [8]. By introducing the co-rotating coordinate $\Phi = \theta + \Lambda z$, where $\Lambda$ is the angular velocity of the scalar RSAB, it turns out, that the only possibility for the vectorialisation procedure to conserve the self-accelerating character of RSABs, is to choose $\boldsymbol{u} = (\boldsymbol{x} \pm i\boldsymbol{y})/\sqrt{2}$, i.e., that in order to maintain their self-accelerating character, vector RSABs must possess circular polarization.

For paraxial vector RSABs, moreover, we calculate their spin (SAM) and orbital (OAM) angular momenta, according to the usual separation $\boldsymbol{J} = \boldsymbol{r}\times(\boldsymbol{E}\times\boldsymbol{B}^*) = \boldsymbol{L} + \boldsymbol{S}$ [9]. Our calculations reveal, that while the global (i.e., integrated) SAM gives a result consistent with the vector beam being circularly polarized (i.e., the helicity of the beam is $\sigma = \pm 1$), the SAM density shows regions of negative value, thus meaning a local inversion of the helicity axis. The OAM, instead, contains, as expected, both intrinsic and extrinsic components. The former is related to the intrinsic OAM carried by Bessel beams, and both its longitudinal and transverse component are proportional to the standard spin-orbit interaction term $m(m+\sigma)$. The latter, on the other hand, has a hybrid nature, since the transverse component of the extrinsic OAM depends only on the angular velocity $\Lambda$ of the RSAB, as it should be for a field rotating about an axis, while its longitudinal component is proportional to the product between the intrinsic OAM carried by Bessel beams composing the RSAB, and its actual angular velocity, i.e., $m\Lambda$.

In conclusion, we have shown, how vector RSABs only retain their self-accelerating character, if they possess circular polarization. Our work constitutes a useful toolbox for experimentally investigating focused RSABs, and to study their interaction with matter and manipulation of dielectric particles.

**References**

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