Shearing Interferometry Using Spin-Orbit Coupling Devices

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Abstract: We present a new approach for performing common-path shearing interferometry using geometric phase introduced via spin-orbit coupling devices. Having a linearly polarized incident wavefront, the shearing mechanism relies on a couple of subsequent identical spatially-varying axis birefringent plates, e.g. a pair of Q-plates, that write opposite geometric phases on the two circularly polarized components, thus enabling almost any tailored directional derivative while securing a compact and robust layout.

1. Introduction

Both in science and engineering, retrieving the phase of an optical field is of great importance for several applications, such as optical metrology or microscopy. While there are a number of methods to implement this retrieval, we are particularly interested in self-referenced approaches that measure a derivative of the field, such as shearing interferometers, varying transmission filters, and Shack-Hartman sensors. The later, although widely popular, are limited by the discretization imposed to the field, the sampling geometry, and the limited range of slopes that can be measured [2,3].

In shearing interferometry, two replicas of the test beam are made to interfere after being displaced laterally (lateral shearing), resized differently (radial shearing), or mutually rotated (azimuthal shearing) [3]. This has an effect similar to the calculation of the spatial directional derivative of the field. There are two main limitations for this method; first, a compromise must be made between an accurate approximation to the derivative and a sufficiently strong signal; and second, in practice only lateral shearing is robust and compact, since the simplicity and robustness of the shearing devices is lost for other shearing geometries. Hence, we propose a novel methodology, based on the use of Spatially Varying-Axis birefringent Plates (SVAPs), to implement a robust and compact shearing interferometer capable of taking a range (both in magnitude and direction) of non-uniform directional derivative distributions.

2. Geometric Phase Shearing Interferometry

A half-wave plate not only reverses the handedness of circularly polarized light but also provides it with a phase dependent on its fast axis direction and with a sign corresponding to its incoming handedness. A SVAP is precisely a half-wave retardation mask whose fast axis orientation angle $\Theta(x,y)$ varies spatially. This means that a circularly polarized beam, after traversing the mask, would change of handedness and acquire a geometric phase $e^{\pm i\,2\Theta(x,y)}$, the sign depending on the incident polarization handedness. This is precisely the basis for the shearing: a slowly varying axis direction would result in inversed local phase ramps for both circular polarizations, deflecting locally each polarization component in opposite ways after the plate by an amount

$$\boldsymbol{\epsilon}(x,y) = \pm z\, k^{-1}\nabla_{\perp}\Theta(x,y) \qquad (1)$$

where z is the propagation distance, k is the wavenumber, and $\nabla_{\perp}$ the gradient along the transversal directions to the optical axis, i.e. x and y. A second SVAP, identical to the first one, then introduces opposite phases which largely (or fully in some geometries like lateral shearing) erases the phase written by the first plate, rectifying the components but leaving them displaced with respect each other. Finally, a linear analyzer is used to make them interfere. For a separation between SVAPs of ζ and a test field $E(x,y)$, by selecting the orientation of the analyzer, ψ, one can control the relative phase between each replica, obtaining

$$I(\psi,\zeta) \approx |E|^2\cos^2\psi + \frac{\zeta}{k}|E|^2\nabla\Theta\cdot\nabla\mathrm{Arg}(E)\sin 2\psi + \frac{\zeta^2}{4k^2}\{[4|\nabla\Theta\cdot\nabla E|^2 + (\nabla^2\Theta)^2|E|^2]\sin^2\psi + 2\nabla^2\Theta\,(\nabla\Theta\cdot\nabla|E|^2)\}. \qquad (2)$$

Traditionally for phase-dominated fields, lateral shearing interferometers (for which $\nabla^2\Theta = 0$), work with the third term, setting $\psi = \pi/2$, enabling the retrieval of the phase directly; other common configurations require the addition or subtraction of at least 2 different analyzer angles. The optimal choice of ψ is the one that makes the second term comparable to the first. By also measuring at $\psi = 0$ we can recover the first term to subtract it. In cases where the intensity is constant, this extra measurement is not necessary.

3. Experiments and Results

To illustrate the functionality of this technique we started with the basic case of lateral shearing (see Figure 1.A), and later explored the radial configuration. The SVAPs we fabricated for lateral shearing, which we call Λ-plates, have a fast axis that rotates in the transverse plane along the x direction, the orientation angle Θ increasing (or decreasing) linearly with x from 0 to π over a distance Λ, representing the spatial period of the plate. Λ-plates, as well as Q-plates, are transparent plates with uniform controllable birefringence, designed and manufactured by the group of Marrucci and Piccirillo [4]. We chose to use them given their efficiency and flexibility. Figure 1.B, shows interferograms for the SLM displaying a cylindrical lens of focal length $f_c = 400\ mm$, whose axis is oriented with respect to the x direction, at different angles. The input's shape is preserved for $\psi = 0$, but not for $\psi = \pi/2$, since the square of the derivative gives two lobes that rotate and become fainter. To complement the lateral shearing test, we present also the measurements of the weak phase introduced by a Q-plate of charge $q = 1$. Finally, we present the results of radial shearing implemented by using Geometric Phase Lenses (GPLs).

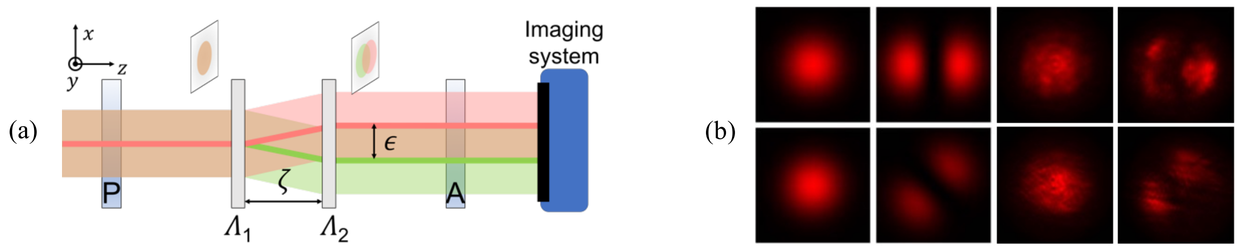

Figure 1: (a) Lateral shearing apparatus: an incoming beam after passing through a linear polarizer P, goes through the two Λ-plates separated by a distance ζ, after which an analyzer is used and the resulting interferogram is recorded at the camera. (b) Lateral shearing interferograms for light focused by a cylindrical lens of focal length, $f_c = 400\ mm$, with different orientations, from top to bottom: 0 and $\pi/4$ with respect the x direction. The first and third columns are the theoretical and experimental profiles for $\psi = 0$, second and fourth show the same for $\psi = \pi/2$.

4. Conclusion

We demonstrated that polarization-based Geometric Phase Shearing Interferometry (GPSI) can be efficiently implemented using a pair of identical geometrical phase optical elements, such as SVAPs. Although we only explored radial and lateral geometries experimentally, the mechanism can be extended to many more geometries. Our device is more compact than all traditional shearing systems. A strength of this technique is its large dynamic range, since it depends on the distance between plates, and more importantly on the capability of tailoring derivatives not only globally (lateral, radial and azimuthal), but also locally in both magnitude and direction, since the distribution can be shaped at will. This technique could find important applications in several types of metrology, e.g., an azimuthal derivative could be used to measure the orbital angular momentum of a beam.

5. References

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