The conservation of helicity in a chiral medium

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Abstract: We consider the energy and helicity densities of circularly polarised light within a chiral medium, and introduce a definition of the helicity density which produces a helicity of $\pm\hbar$ per photon for right and left circular polarisation, respectively. We examine the helicity continuity equation, and show that this form of the helicity density is required for consistency with the dual symmetry condition of a chiral medium with a constant value of ε/μ. We extend the results to higher order in the chirality parameter, and establish an exact relationship between the helicity and energy densities of a wave of general form inside a chiral medium. By examining the group velocity of the waves, we further propose exact expressions of the helicity and energy densities for circular polarised plane waves in a chiral medium.

The free-space Maxwell equations are invariant under duality transformation, reflecting the conservation of helicity in the absence of charge [1]. Treating the effects of charges in a medium using macroscopic electrodynamic extends this dual symmetry to lossless chiral media, as long as the ratio ε/μ remains constant throughout the material [2].

The helicity of right and left circular polarised light in vacuum is $\pm\hbar$ per photon [1]. We consider the propagation of light from vacuum into a lossless chiral medium, characterised by ε, μ, and the chirality parameter β, further imposing $\varepsilon/\mu=\varepsilon_0/\mu_0$ so that helicity is conserved at the interface. Since helicity and energy [3] are conserved, it follows that the ratio of helicity density to energy density remains unchanged from the vacuum value, and that the helicity of each photon is again given by $\pm\hbar$ for the two polarisations.

To $\mathcal{O}(\beta^n)$, we show that a helicity density of the form

$$h_n = \frac{1}{2}\left(\sqrt{\frac{\varepsilon}{\mu}}\mathbf{A}\cdot\mathbf{B} - \sqrt{\frac{\mu}{\varepsilon}}\mathbf{C}\cdot\mathbf{D}\right) + \sqrt{\varepsilon\mu}\beta\ w_{n-1}, \tag{1}$$

where w_{n-1} is the energy density to $\mathcal{O}(\beta^{n-1})$, is required to produce this value of helicity per photon in a chiral medium. We further demonstrate that only a helicity density of the form (1) allows us to express the local conservation of helicity in a chiral medium by way of a continuity equation $\partial_t h_n + \nabla\cdot\mathbf{v} = 0$, where $\mathbf{v}$ is the helicity flux density [2].

We calculate the group velocity of the right and left circular polarised light in a chiral medium as $v_g^{\pm} = 1/\sqrt{\varepsilon\mu}(1\pm\beta k)^2$, and use explicit forms of the energy density up to $\mathcal{O}(\beta^3)$ to show that this velocity is consistent with the from of the helicity density (1). From this, we propose exact expressions to all orders in β of the helicity and energy densities of right and left circular polarised light inside a chiral medium.

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[2] K. Van Kruining, J. B. Götte, "The conditions for the preservation of duality symmetry in a linear medium" *J. Opt* **18,** 085601 (2016).
[3] S. M. Barnett, R. P. Cameron, "Energy conservation and the constitutive relations in chiral and non-reciprocal media" *J. Opt* **18,** 015404 (2016).