Entangling OAM states using diffraction

Aaron Z. Goldberg* and Daniel F. V. James
University of Toronto, 60 St. George St., Toronto, ON, Canada, M5S 1A7
*goldberg@physics.utoronto.ca

Abstract: Orbital angular momentum (OAM) is a robust carrier of quantum information, making entanglement between OAM states a crucial resource for quantum technologies. We show that entangled OAM states can be generated using simple diffraction of non-entangled light beams. We further give the necessary and sufficient conditions for converting nonclassical input light into entangled output light. These results can be extended to all of Fourier optics.

Many exciting new proposals rely on multidimensional quantum entanglement. The Laguerre-Gaussian (LG) modes of an electromagnetic field, which comprise a discrete infinite-dimensional Hilbert space and possess well-defined orbital angular momentum (OAM), have gained prominence as a platform for realizing these proposals. Entanglement between LG modes is normally generated using parametric down-conversion; unfortunately, this method is nondeterministic, and can require materials with prohibitively large nonlinearities. We propose that OAM entanglement instead be generated by inducing photon-photon interactions via beam splitters. We show that diffraction can achieve beam-splitter-like transformations, thus enabling entanglement generation via diffraction [1].

The diffraction of an electric field E is well-described by an impulse response function $h(\boldsymbol{r},\boldsymbol{r_0})$ [2]:

$$E_{\text{out}}(\boldsymbol{r}) = \int d\boldsymbol{r_0}\, h(\boldsymbol{r},\boldsymbol{r_0}) E_{\text{in}}(\boldsymbol{r_0}). \tag{1}$$

We decompose the incoming and outgoing fields using a complete orthonormal basis of LG modes $\phi_m(\boldsymbol{r})$. These modes transform linearly, in exactly the same manner as a multiport beam splitter [1]:

$$\phi_{\text{m}}(\boldsymbol{r}) \rightarrow \sum_n U_{mn}\phi_n(\boldsymbol{r}) \tag{2}$$

$$U_{mn} = \int d\boldsymbol{r}\, d\boldsymbol{r_0}\, \phi_n^*(\boldsymbol{r}) h(\boldsymbol{r},\boldsymbol{r_0}) \phi_m(\boldsymbol{r}) \tag{3}$$

Just as beam splitters can entangle bosonic modes in the Hong-Ou-Mandel effect, so too can multiport beam splitters generate multidimensional entanglement. This means that diffraction can be used to entangle the LG modes $\phi_m(\boldsymbol{r})$.

Entanglement can only be generated at a beam splitter if the input state is nonclassical. There has been some recent effort in distinguishing the types of nonclassical states that can and cannot be used for entanglement generation [3,4]. We find the necessary and sufficient input states that will generate entanglement in any specific subset of output modes. The only states that generate no entanglement are coherent states that are equally squeezed in every mode that couples to any of the output modes in the specific subset [1]. All other nonclassical input states will generate entanglement at any multiport beam splitter, including electromagnetic field diffraction.

As an example, consider using diffraction of an LG mode incident on the $z = 0$ plane to achieve some desired transformation $\phi_0(\boldsymbol{r}) \rightarrow \phi_m(\boldsymbol{r}) + \phi_n(\boldsymbol{r})$. Denoting the two-dimensional Fourier transform by $\mathcal{F}$, we can use the convolution theorem to find the appropriate impulse response function $h(\boldsymbol{r}-\boldsymbol{r_0}) = \mathcal{F}^{-1}\left(\frac{\mathcal{F}(\phi_m+\phi_n)}{\mathcal{F}(\phi_0)}\right)$. This immediately yields entanglement between modes m and n for any nonclassical state input into mode 0.

Arbitrary impulse response functions are constructible by varying the diffractive setup; a wide range of beam splitter transformations can be tailored using diffraction. Entangled LG modes can thus be easily generated using Fourier optics, paving the way for deterministic OAM entanglement.

References

[1] A. Z. Goldberg and D. F. V. James, *Entanglement generation via diffraction*, in prep.
[2] J. W. Goodman, *Introduction to Fourier Optics*, 3rd ed. (Roberts & Company Publishers, 2005).
[3] Z. Jiang, M. D. Lang, and C. M. Caves, "Mixing nonclassical pure states in a linear-optical network almost always generates modal entanglement," *Phys. Rev. A* **88**, 044301 (2013).
[4] A. Z. Goldberg and D. F. V. James, "Nonclassical mixed states that generate zero entanglement with a beam splitter," *Journal of Physics A: Mathematical and Theoretical* **51**, 385303 (2018).