Radiation of Twisted Photons by Classical Currents

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Abstract: The general formula for the probability of radiation twisted photons by classical currents is derived. The theory of undulator radiation of twisted photons is developed. The infrared asymptotics of probability of radiation of twisted photons in an arbitrary QED process is found. The generation of twisted photons by axially symmetric bunches of particles is described. Several selection and sum rules for the radiation of twisted photons are proved.

The twisted photons are the solutions of the free Maxwell equations with definite the energy k_0, the longitudinal projection of momentum k_3, the projection of the total angular momentum m, and the helicity s (see, e.g. [1]). These solutions constitute a complete set and any free electromagnetic field can be represented as a collection of twisted photons. Such a representation of the electromagnetic field reveals its properties that are not evident in the standard plane-wave form. At present, sufficiently narrow in quantum numbers wave packets of twisted photons and their superpositions are used in fundamental science and technology (see for review, [2]). Recently, we have derived the general formula for the probability of radiation of twisted photons by classical currents [3]:

$$dP(s,m,k_3,k_\perp)=\Big|\sum_{l=1}^{N} e_l\int d\tau e^{-i[k_0 x_l^0(\tau)-k_3 x_{3l}(\tau)]}\Big\{\frac{1}{2}\big[\dot{x}_{+l}(\tau)a_-(s,m,k_3,k_\perp;\mathrm{x}_l(\tau)) + \dot{x}_{-l}(\tau)a_+(s,m,k_3,k_\perp;\mathrm{x}_l(\tau))\big]+\dot{x}_{3l}(\tau)a_3(m,k_\perp;\mathrm{x}_l(\tau))\Big\}\Big|^2\left(\frac{k_\perp}{2k_0}\right)^3\frac{dk_3dk_\perp}{2\pi^2}, \qquad (1)$$

where e_l are the particle charges, $a_\pm, a_3$ are the mode functions of twisted photons, and $x_\pm(\tau), x_3(\tau)$ specify the trajectory of a charged particle.

Employing this formula, we develop a general theory of undulator radiation of twisted photons in both the dipole and wiggler regimes for an arbitrary position of the twisted photon detector. The explicit formulas for the average number of twisted photons generated by undulators are obtained. The symmetry property of the average number of twisted photons produced by a charged particle moving along a planar trajectory is found. Several selection rules are established. In particular, it is found that for the forward radiation of twisted photons by a planar undulator n+m is an even number, where n is a harmonic number. The average number of twisted photons produced by the undulator and detected off the undulator axis is a periodic function of m in a certain spectral band of the quantum numbers m.

Then in [4] we analyze in detail the infrared behavior of formula (1). The infrared asymptotics of radiation of photons in a vacuum is known to be universal for any QED process and does not depend on spin of radiating particles (see, e.g., [5]). It is described by the radiation produced by the classical current of charged particles moving uniformly along straight lines with a break. We find the exact formula for the radiation probability of twisted photons and establish the symmetry property of the radiation probability of twisted photons

$$dP(s,m,k_3,k_\perp)=dP(-s,-m,k_3,k_\perp), \qquad (2)$$

that holds for any QED process in the far infrared. We prove the general selection rules for the radiation and absorption of twisted photons that allow one to modulate the one-particle radiation probability and produce the twisted photon radiation with desired properties. The effect of a finite width of a particle bunch on radiation of twisted photons is studied in [6]. We find several sum rules for the radiation of twisted photons by axially symmetric bunches of particles.

References

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