Total Internal Reflection as 'Phase Transition' of Light

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Abstract: Evanescent waves have non-trivial characteristics such as superluminal behaviour and transverse spin, which ordinary propagating waves do not have [1]. Bekshaev *et. al.* introduced the concept of complex angle of refraction to deal with evanescent waves [2]. We analyse the behaviour of the imaginary part of the complex to find that it changes discontinuously angle at the condition of total internal reflection. We regard this imaginary part of the angle as an 'order parameter' in order to characterise 'phase' of light. The non-trivial behaviours of evanescent waves including superluminality, transverse spin, and helical/chiral edge of light can be described by the order parameter.

1. 'Phase transition' of light near interfaces

One way to generate evanescent waves is total internal reflection (TIR). The condition of TIR is $\sin\theta_c = n$. Here, $n \equiv n_2/n_1$ is the relative index of refraction. Figure 1 shows the imaginary part of angle of refraction as a function of the relative index and the angle of incidence. We can clearly see the discontinuous transition of ψ_2 from zero to non-zero value on the black solid line. We call this discontinuous transition 'phase transition' of light.

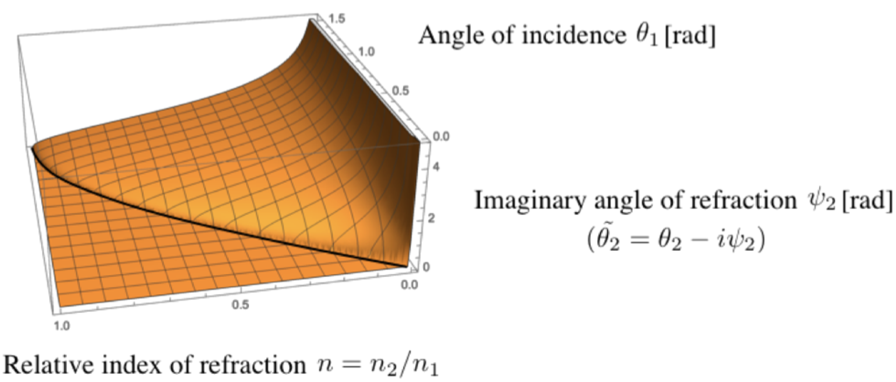

Fig. 1. Discontinuous changes of 'order parameter' ψ_2. This figure is the surface plot of the imaginary part of the angle of refraction ψ_2 as a function of relative index n and angle of incidence θ_1.

2. Extraordinary quantities characterised by order parameter ψ_2

With the appearance of the imaginary angle ψ_2, light changes its characteristics significantly.

• *Superluminal propagation.* —This means that the group velocity in the in-plane direction locally exceeds the speed of light in free space ($v_{g,x} \equiv d\omega/dk_x = c \cdot \cosh\psi_2 > c$).

• *Transverse spin.* —We can define the spin vector of light by $\vec{S} \equiv \frac{1}{2}\mathrm{Im}\left(\epsilon\vec{E}^* \times \vec{E} + \mu\vec{H}^* \times \vec{H}\right)$, which represents the rotation direction of polarisation. This vector does not have the transverse components in case that $\psi_2 = 0$, while the transverse components emerges in case that $\psi_2 \neq 0$. For example, when the incident light is p-polarised, the transverse y-component of $\vec{S}$ is non-zero ($S_y \propto -\sinh\psi_2 \cosh\psi_2 \neq 0$).

• *Chiral/helical edge flow.* —Spin vector has another important physical meaning. It can be regarded as *helicity flow*, for spin and helicity satisfy the continuity equation: $\nabla \cdot \vec{S} + \frac{\partial h}{\partial t} = 0$. Here, we define *helicity* $h \equiv \frac{\epsilon\mu}{2}\mathrm{Im}(\vec{E}^* \cdot \vec{H})$. *Helical edge flow* (helicity flow in transverse y-direction) is generated near the interface when incident light is p-polarised, while *chiral edge flow* (energy flow in transverse y-direction) is generated when incident light is circularly-polarised [1]. Neither of the flows exist for the ordinary propagating light.

[1] K. Bliokh, *et. al.*, "Extraordinary momentum and spin in evanescent waves." *Nature communications* **5**, 3300 (2014).

[2] A. Bekshaev, *et. al.*, "Mie scattering and optical forces from evanescent fields: A complex-angle approach." *Optics express* **21**, 7082 (2013).