Arbitrary High-Dimensional *X*-Gates of a flying Quqit

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Abstract: High-dimensional quantum system can encode more quantum information than their two dimensional counterparts. While the increased information capacity has advantages in quantum applications, manipulating these systems has been challenging. Here we show a method to perform deterministic arbitrary high-dimensional Pauli X gates of single photons carrying orbital angular momentum. The proposed experimental setups only use two basic optical elements such as mode sorters and mode shifters and thus could be implemented in any system where these experimental tools are available. Importantly, the number of involved interferometers scales logarithmically with the dimension, which is important for practical implementation.

High-dimensional quantum systems allow for encoding, transmitting and processing more than qubits system. The exploitation of large alphabets in quantum communication protocols can significantly improve their capacity [1] and security [2]. However, performing well-defined manipulations in multilevel systems is significantly more challenging than for qubits. Laguerre-Gaussian modes of light, carrying orbital angular momentum (OAM) [3], represent an alternative to polarization that has been applied a lot in high-dimensional quantum domains [4-6]. The X gate in high-dimensional Hilbert spaces takes the form of a cyclic permutation of the computational basis vectors. For a fixed basis in a d-dimensional space the cyclic transformation transforms each basis state into its nearest neighbour in a clockwise manner with the last state being transformed back to the first one [7]. While efficient methods for realizing a four-dimensional cyclic transformation in both classical and quantum realms have been experimentally demonstrated [7-9], for an arbitrary dimension such methods are still missing.

Here we present setups of X gates for arbitrary d-dimensional qudits represented by the OAM of single photons. When developing the general method we took inspiration from designs generated by the computer program MELVIN [10]. Schemes produced by the method can be implemented in the laboratory using accessible optical components. The setups employ only two basic elements: holograms and OAM beam splitters (OAM-BSs) introduced by Leach et al. [11]. Importantly, the number of OAM-BSs (which are interferometric devices) scales logarithmically with the dimension of the cycle, which is relevant for their experimental implementation. A cyclic transformation in a given d-dimensional space is defined for a specific basis, but also for other sets of states without adaptation of the experimental setup. The structure of the setups generated by our method is very symmetric. Consequently, if the photon has a sufficiently large coherence length, the original setup can be considerably simplified.

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