# Are high-dimensional quantum states robust to noise?

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**Abstract:** The Werner state is the most common mathematical construct used to model the noise present in a quantum information system. This state considers the ideal, maximally entangled state together with uncorrelated noise. When this state is used to model noise on states in high dimensions, one reaches the conclusion that increasing the dimension increases the state's robustness to noise. However, in realistic experiments, where detector imperfections are considered, the reverse conclusion is found: increasing the dimension does not always increase the robustness to noise. In this sense, noise in the state and noise on the detector must be considered independently to accurately predict the performance of high-dimensional states. Here we present an operational study of high-dimensional entanglement in the presence of noise by considering multi-photon pairs and modelling realistic detectors. While high-dimensional states are robust to noise in the state, they are not robust to noise on the detector.

Previously, Werner states [1] have been used in theoretical models to represent high-dimensional entangled states with uncorrelated noise. Such a system models noise in the state but does not consider dimension-dependent sources of noise, such as detector dark counts. When one considers entanglement or non-locality (CGLMP inequality) in high dimensions, the Werner state shows an increased robustness to state noise [2]. Here we develop a model that incorporates noise into the measurement operators in terms of operational parameters in experiment, as well as noise present in the state. In our model, we consider a density matrix corresponding to a photonic high-dimensional entangled state with multi-photon pairs and measurements that include parameters for detector dark counts and total collection efficiency. We derive an expression for the generated photon pair state that includes contributing terms from multi-photon pair events and calculate the conditions for violation of the CGLMP inequality with the modified measurements. Equation (1) shows how the value of the CGLMP parameter $I_0$ (ideal noiseless case) is modified dependent on the dimension $d$, pair generation amplitude $p$, and noise-efficiency ratio $\mu$.

$$I(d,p,\mu) = \frac{I_0}{\left[1+|p|^2\left(2+\frac{2}{d}\right)\right](1+2d\mu)+\mu^2 d^2/|p|^2}. \qquad (1)$$

For a state to show non-locality, the measured value of $I$ must be greater than 2. In figure 1 we show $I$ plotted against various values of $p$, $d$, and $\mu$. Figure 1 (b) shows that as the $\mu$ is increased, the maximum dimension in which non-locality is observed decreases. From these results, we can conclude that the degree of high-dimensional entanglement depends on a complex interplay between noise in the state and detection system.

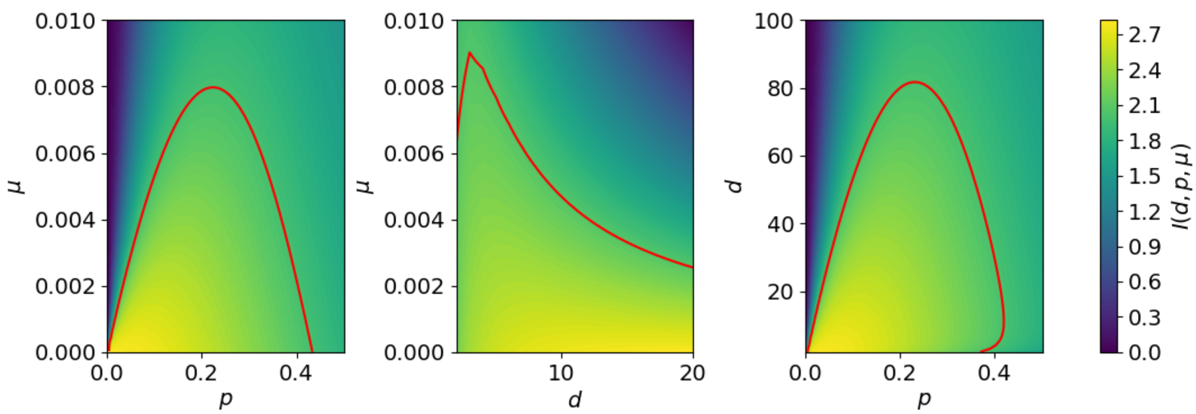


Fig.1(a). I vs $\mu$ and p for d=10 (b) I vs $\mu$ and d for p=0.355 (c) I vs p and d for $\mu$=0.001

[1] R. Werner, *et al.*, "Quantum states with Einstein-Podolsky-Rosen correlations admitting a hidden-variable model," *Phys. Rev. A* **40,** 4277 (1989).

[2] D. Collins, *et al.*, "Bell inequalities for arbitrarily high-dimensional systems," *Phys. Rev. Lett.* **88,** 040404 (2002).