The modified interferometer as twisting operations on spatial modes

Yoko Miyamoto and C T Samlan

Department of Engineering science, The University of Electro-communications, Chofu, Tokyo, Japan – 180-8585

Author e-mail address: (yoko.miyamoto@uec.ac.jp)

Abstract: We propose a pair of twisting operators to describe the actions of a modified Sagnac interferometer on general spatial modes.

1. Introduction

It has been shown that a modified Sagnac interferometer with position and momentum shifts in orthogonal directions can introduce a vortex in a Gaussian input beam [1]. This system can be used to generate broadband vortex and vector beams, as well as provide a new platform for manipulations of spatial modes. In this paper we develop a general description of this system in the form of "twisting operators" on spatial modes.

2. Twisting operators

The interferometer works by displacing the two copies of the input beam relative to each other by $2\Delta y$, introducing opposite transverse momentum $\pm\Delta k$ in the orthogonal direction, and superposing them with relative phase π. Calculating to first order in Δy and Δk, the output for an input complex amplitude $f(x, y)$ can be given by

$$e^{\pm i\Delta kx}\left(1+\Delta y\frac{\partial}{\partial y}\right)f-e^{\mp i\Delta kx}\left(1-\Delta y\frac{\partial}{\partial y}\right)f\sim\pm 2i\Delta k\left(x\pm\frac{\Delta y}{i\Delta k}\frac{\partial}{\partial y}\right)f. \qquad (1)$$

For Gaussian input $f=e^{-(x^2+y^2)/2}$, the output becomes $\pm 2i\Delta k\left(x\pm i\frac{\Delta y}{\Delta k}y\right)e^{-(x^2+y^2)/2}$. Setting $\Delta y=\Delta k$ (here we use dimensionless notation where a scaling length w gives the actual length $w\Delta y$ and wavenumber $\Delta k/w$) produces a canonical vortex. Omitting overall phase and amplitude factors, the twisting operators in equation (1) can be written as

$$\hat{T}_{\pm}=x\pm\frac{1}{i}\frac{\partial}{\partial y}. \qquad (2)$$

The operators can be rewritten in terms of the ladder operators $\left(\hat{a}_x,\hat{a}_x^\dagger\right)$ and $\left(\hat{a}_y,\hat{a}_y^\dagger\right)$ for 1-dimensional Hermite Gaussian functions as

$$\hat{T}_{\pm}=\left\{\left(\hat{a}_x\mp i\hat{a}_y\right)+\left(\hat{a}_x^\dagger\pm i\hat{a}_y^\dagger\right)\right\}/\sqrt{2}. \qquad (3)$$

The operators $\hat{a}_x\mp i\hat{a}_y$ and $\hat{a}_x^\dagger\pm i\hat{a}_y^\dagger$ are associated with manipulations on angular momentum[2], providing insight into the properties of $\hat{T}_{\pm}$.

Figure 1 shows operations of the twisting operators on some lower-order LG_p^l mode inputs. Operation of $\hat{T}_+$ on LG_0^1 produces a superposition of LG_0^2 and LG_0^0, showing simultaneous up and down operations on angular momentum. $\hat{T}_-LG_0^1$ on the other hand produces only LG_1^0, which can be linked to the constraint on radial mode number.

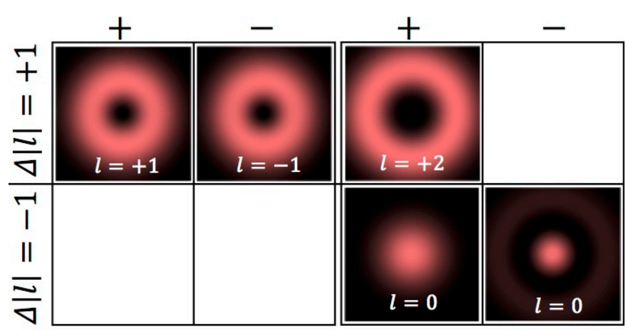

Fig. 1: Twisting operations on LG_0^0 (left) and LG_0^1 (right) inputs.

3. Summary

The twisting operators provide a concise expression of the modified interferometer to help explore its actions on general inputs. The results indicate selection rules for radial and azimuthal mode numbers.

[1] D. N. Naik, *et al.* "Ultrashort vortex from a Gaussian pulse–An achromatic-interferometric approach" *Sci. Rep.* **7,** 2395 (2017).

[2] G. Nienhuis and L. Allen. "Paraxial wave optics and harmonic oscillators" *Phys. Rev.* A **48,** 656 (1993).