Majorana Representation of Structured-Gaussian Beams

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Abstract: Through the underlying SU(2) group structure of structured-Gaussian beams, a reduced spherical phase space can be defined, in which a beam is represented by a collection of points known as the Majorana constellation. This stands as the proper generalization of the modal Poincaré sphere to higher orders. Moreover, an invariance to an astigmatic transformation is translated into a rotational symmetry of the constellation and gives way to continuous or quantized geometric phases.

Structured-Gaussian (SG) beams are solutions to the paraxial wave equation for which the intensity profile remains invariant (up to a scaling factor) upon propagation. Among these are the well-known Hermite-Gauss (HG) and Laguerre Gauss (LG) beams which have been the subject of extensive research. Other known examples are the generalized Hermite-Laguerre (HLG) beams obtained via astigmatic transformations from HG or LG modes [1]. The HG, LG and HLG modes connected through astigmatic transformations are customarily represented as points on the surface of the modal Poincaré sphere (MPS) [2,3]. However, this representation is restricted to HLG beams and different spheres are needed to differentiate between the modes.

Using the operator formalism, SG beams can be shown to be analogous to quantum angular momentum through Schwinger's oscillator model [4,5]. The total order N and the azimuthal index l of LG modes play the role of quantum numbers and SG beams are obtained by restricting the space to modes with same N. Given this mathematical similarity, spin-coherent states, corresponding to extremal ($l=N$) HLG modes, can be used to define a Q function on the surface of a sphere. The zeros of this phase-space representation define the Majorana constellation [5,6], a set of N points that uniquely define an SG beam. Figure 1 shows an icosahedron beam as an example. Therefore, the Majorana representation stands as the proper generalization of the MPS to higher order modes.

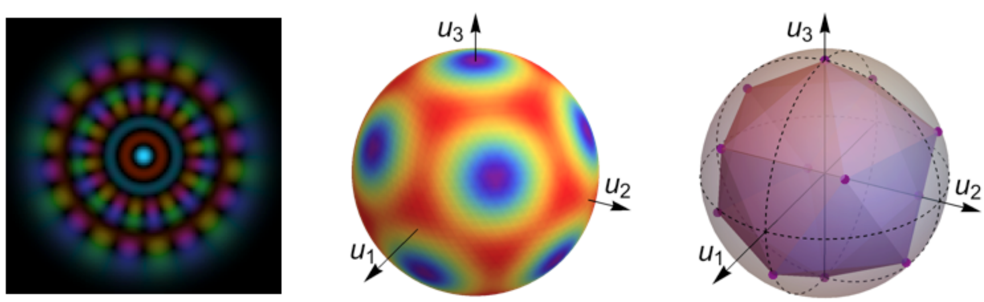

Figure 1. Icosahedron beam. (From left to right) Intensity distribution with the phase coded as a hue, Majorana constellation and Q function.

The constellation also provides information about its angular momentum content and invariances to specific astigmatic transformations through its rotational symmetries. The rotational symmetries give way to (continuous or quantized) geometric phases which can be determined solely from the constellation. This allows the design of highly symmetric beams, such as those given by platonic solids (see Fig. 1) and with specific quantized geometric phases [5]. Furthermore, the (continuous or discrete) geometric and Gouy phases can be measured by a non-interferometric method [7,8].

References

[1] E. G. Abramochkin and V. G. Volostnikov, "Generalized Gaussian beams," *J. Opt. A: Pure Appl. Opt.* **6**, S157 (2004).
[2] M. J. Padgett and J. Courtial, "Poincaré-sphere equivalent for light beams containing orbital angular momentum," *Opt. Lett.* **24**, 430 (1999).
[3] G. F. Calvo, "Wigner representation and geometric transformations of optical orbital angular momentum spatial modes," *Opt. Lett.* **30**, 1207 (2005).
[4] M. R. Dennis and M. A. Alonso, "Swings and roundabouts: optical Poincaré spheres for polarization and Gaussian beams," *Phil. Trans. R. Soc. A* **375**, 20150441 (2017).
[5] R. Gutiérrez-Cuevas and M. A. Alonso, "Modal Majorana sphere and hidden symmetries of structured-Gaussian beams," arXiv:1901.06987 [physics.optics] (2019).
[6] E. Majorana, "Atomi orientati in campo magnetico variabile," *Il Nuovo Cimiento* **9**, 43 (1932).
[7] M. A. Alonso and M. R. Dennis, "Ray-optical Poincaré sphere for structured Gaussian beams," *Optica* **4**, 476 (2017).
[8] T. Malhotra, R. Gutiérrez-Cuevas, J. Hassett, M. Dennis, A. Vamivakas, and M. Alonso, "Measuring Geometric Phase without Interferometry," *Phys. Rev. Lett.* **120**, 233602 (2018).