Skyrmion structure in vector beams

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Abstract: Vector beams have both a spatially varying amplitude and polarization, for light, or spin direction for electrons. Associated with these we find a topological feature, the skyrmion number, that it is robust against any deformation during propagation. The simplest way to map out this feature is to use the associated skyrmion field. This is a transverse or divergence-less field and so has neither sources nor sinks. Mapping out the skyrmion field for these beams reveals constraints on the manner in which vector beams can be manipulated.

The form of a vector beam, either for a single electron or an optical beam, can be represented in terms of a local and normalized spin state:

$$|\Psi(\boldsymbol{r},t)\rangle = \frac{\varphi_0^{\ell+}(r,t)|P\rangle + \varphi_0^{\ell-}(\boldsymbol{r},t)|V\rangle}{\sqrt{\left|\varphi_0^{\ell+}(\boldsymbol{r},t)\right|^2 + \left|\varphi_0^{\ell-}(\boldsymbol{r},t)\right|^2}} \,. \qquad (1)$$

Here $\varphi_0^{\ell+}$ *and* $\varphi_0^{\ell-}$ are typically a pair of Laguerre-Gaussian beams with the same beam waist but different winding number ℓ. The states $|P\rangle$ and $|V\rangle$ represent either any two orthogonal spin directions for the electron or any two orthogonal directions on the Poincaré sphere for a light beam.

Skyrmions were introduced by Skyrme in 1962 [1] as a topological soliton for the pion field, aimed at explaining the stability of hadrons. Although this idea is not part of the mainstream particle theory, it has been adopted widely in condensed matter systems. In particular, magnetic skyrmions are quasi particles, point like regions of magnetization, which have been theoretically predicted [2] and experimentally observed [3]. Skyrmions are characterized by the topological integer *n*, the skyrmion number, which can be evaluated as a surface integral of the form

$$n = \frac{1}{4\pi}\int \boldsymbol{M}\cdot\left(\frac{\partial \boldsymbol{M}}{\partial x}\times\frac{\partial \boldsymbol{M}}{\partial y}\right)dxdy, \qquad (2)$$

where $\boldsymbol{M}$ is the unit vector in the local magnetization field.

We can obtain the analogous quantity for our electron or optical beam by replacing $\boldsymbol{M}$ with local spin direction or Poincaré vector direction respectively. For the state given in (1) we find the simple value:

$$n = \ell_+ - \ell_- \,. \qquad (3)$$

We note that this does not depend on the specific orientation of the spin or polarization, indeed performing a global spin or polarization rotation does not change this result.

We can also understand the skyrmion number as the flux of a vector field, the skyrmion field, in the *z* direction. The general form of this skyrmion field is

$$\Sigma_i = \frac{\varepsilon_{ijk}}{2}\,\boldsymbol{M}\cdot(\partial_j\boldsymbol{M})\times(\partial_k\boldsymbol{M}) \,. \qquad (4)$$

It is not difficult to verify that this field is divergence-less, which means that the associated field lines can form loops but have no sources or sinks. This means that the field can neither be created or destroyed and that, for example, the creation of a beam with skyrmion number 1 is accompanied by the creation of a further beam with skyrmion number -1. We note that the skyrmion field, being divergence-less, can be written as a curl of another vector field. We evaluate this quantity and note its similarity to the velocity field in the theory of superfluids [4].

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[3] N. Romming, et al. "Writing and deleting single magnetic skyrmions." *Science* **341**(6146): 636-639(2013).
[4] D. Vollhardt and P. Wölfe "The superfluid phases of helium 3" (Dover, New York, 2013).